Plenary Panel 3

Pandemic Times: Challenges, Responsibilities and Roles for Mathematics and Mathematics Education Communities

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ABSTRACT The Plenary Panel 3 at ICME-14 was especially devoted to the COVID-19 pandemic. Its goal was to review and reflect on the challenges raised by the pandemic, the responsibilities and roles for mathematicians and mathematics educators in this context, and to draw some lessons for the future. This text begins with the presentation of the panel and the four invited panelists. Then each panelist synthesizes her/his contribution to the panel, and we end by some lessons drawn from these contributions and the exchanges between the panelists and with the audience.

Keywords: COVID-19 pandemic; Mathematics; Mathematics Education

1. Introduction

Our lives, our educational systems and our societies have been turned upside down by the COVID-19 pandemic. The goal of Plenary Panel 3, late addition to the congress scientific activities, was to review and reflect on the challenges, responsibilities and roles for mathematicians and mathematics educators in these pandemic times, to identify possible synergies between communities that can assist them in this context, and to draw some lessons for the future. Its co-chairs, Michèle Artigue and Ingrid Daubechies, respectively former president of ICMI and IMU, invited the contributions of four panelists, two mathematicians and two mathematics educators, living and working in very different environments, with a wide range of expertise and experiences.

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The mathematicians were Timothy Gowers from the Collège de France in Paris and from the University of Cambridge in the UK, and Jean Lubuma, then working at the University of Pretoria in South Africa. Gowers is a well-known mathematician, specialist of combinatorics, who was awarded the Fields Medal in 1998. He has long been interested in societal problems and popularization of mathematics, at many levels, and with respect to the Covid-19 pandemic, his writings influenced UK government decisions. Lubuma is an applied mathematician, fellow of the African Academy of Sciences and member of the Academy of Science of South Africa, who has done extensive epidemiological research in Africa, to provide a sound mathematical analysis and elaborate realistic solutions to (re-)emerging diseases such as COVID-19, Ebola, HIV/AIDS, Malaria, etc. that pose a threat to development of the continent.

The mathematics educators were Nelly León Gómez from the Universidad Pedagógica Experimental Libertador (UPEL) in Maturín, Venezuela, and David Wagner, from the University of New Brunswick in Canada. León worked for more than 40 years at UPEL as a teacher educator and researcher in mathematics education. Her responsibilities in the Inter-American Committee on Mathematics Education (IACME) and the Mathematical Education Network of Central America and the Caribbean (REDUMATE) make her especially knowledgeable about the problems generated by the pandemic in Latin America. Wagner is a researcher in mathematics education whose research especially addresses the cultural nature of mathematics and the impact of mathematics teaching practices on individuals and society. Co-editor-in-chief of Educational Studies in Mathematics, he was co-editor of the special issue "Mathematics education in a time of crisis – a viral pandemic", in preparation at the time of ICME-14.

Taking into account the diversity and complementarity of expertise and experience of the four panelists, the co-chairs prepared a specific set of questions for each of them, the exact formulation of which was discussed with them (see sections below). A collective question was added regarding the synergies observed between mathematics and mathematics education in these pandemic times. During the panel session, the co-chairs first presented the panel organization and introduced the panelists; this was followed by the videos recorded by each of the panelists in response to the specific questions asked to them. Next came an interactive phase in which the panelists exchanged views among themselves, with the co-chairs and with the participants attending the session in Shanghai.

Sections 2 to 5 are devoted to the panelists' contributions. We follow the order of their presentation at ICME-14 and each section starts with the questions especially addressed by the panelist. Section 6 draws some main lessons from this panel.

2. Teaching, Learning and Research in Mathematical Sciences in Pandemic Times. *Panelist: Jean Lubuma*

Part of your mathematical research has been focused on questions related to epidemiology. How would you describe the role of mathematicians in this field?

Does the current pandemic present particular challenges? If so, how are your colleagues and yourself dealing with them? Which of these do you find most difficult?

We describe the role of mathematicians in disease epidemiology and highlight some challenges presented by the COVID-19 pandemic. We outline how we dealt with them at the University of Pretoria (UP), and we make a call to the international community to invest in teaching and research in mathematics in developing countries, particularly in Africa that counts the top 10 poorest countries in the world.

2.1. The role of mathematicians in disease epidemiology

The role of mathematicians in the epidemiology of human infectious diseases is to develop, analyze and simulate realistic mathematical models for gaining insight into the disease transmission dynamics and control. The use of mathematics in epidemiology goes far back to the middle of the eighteenth century, with the pioneering work of Bernouilli (1760) on modelling the effectiveness of immunization against smallpox. Two centuries later, Ross (1911), who received the Nobel Prize of Medicine in 1902, presented the first mathematical model for malaria transmission, and showed that the disease could be effectively controlled or eliminated if the malaria vector population is reduced below a certain critical threshold. In a series of seminal papers, Kermack & McKendrick (1927) and Macdonald (1957) further developed and formalized Ross' work into the theory of epidemics and laid the foundation for compartmental modelling. In recent years, there has been a strong focus on the modelling of emerging and re-emerging infectious diseases of public health significance; see Castillo-Chavez et al. (2002).

The threshold theory is expressed in terms of the basic reproduction number, $\mathcal{R}_0$, an important epidemiological threshold quantity, defined as the average number of secondary infections generated by a single infectious individual (during his or her entire infectious period) if introduced into a wholly susceptible population. The mathematical definition and the methodology for practically computing $\mathcal{R}_0$ are due to Diekmann et al. (1990) and van den Driessche & Watmough (2002): $\mathcal{R}_0$ is the spectral radius of the associated next generation matrix, K, of the model being studied. For typical models, the threshold theory is stated in the next theorem that is illustrated in Fig. 1 for $\mathcal{R}_0 = 2$; see Gumel (2021).

Theorem 1: *The disease-free equilibrium is locally asymptotically stable if* $\mathcal{R}_0 < 1$, *and unstable if* $\mathcal{R}_0 > 1$.

It follows that, for a vaccine-preventable disease, a fraction $p > 1 - 1/\mathcal{R}_0$ of susceptible individuals should be immunized (assuming a perfect vaccine) against the disease, to achieve herd immunity.

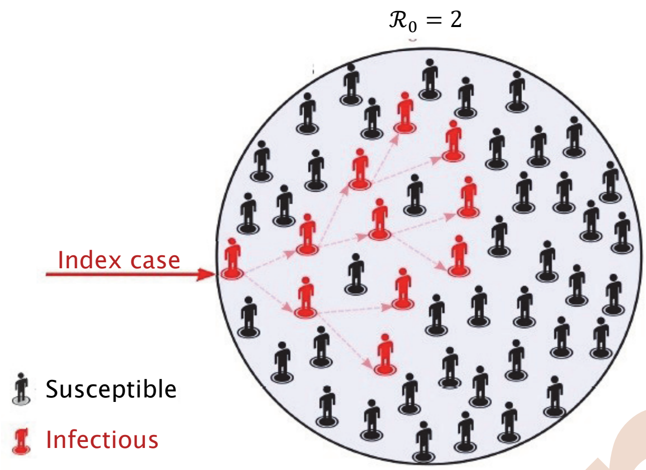

Fig. 1 If $I_{n+1} = K^n I_0$ with I the infective variables and K the (typically positive) next generation matrix, the number of secondary infectious per generation, n, is $\mathcal{R}_0$, the largest eigenvalue of $K \geq 0$. For $\mathcal{R}_0 > 1$, the final state/size of I is $I_\infty = 0$ (epidemic) or $I_\infty > 0$ (endemic).

2.2. Challenges and COVID-19 modelling

Despite the rapid effort made by scientists to isolate the SARS-CoV-2 virus (the causative agent of the COVID-19 pandemic), to sequence it, to develop a diagnostic test and to produce vaccines, numerous unknown and open questions and challenges linger around the COVID-19 pandemic. Some of the challenges are linked to the following points: social-cultural dynamics of transmission, comorbidity *vs* forgotten diseases, new variants of the virus, concerns around vaccines, and lockdown *vs* economy recovery. Regarding the lockdown, the suspension of face-to-face teaching in Africa practically means no teaching and learning since the continent is not well digitalized to offer online teaching, which is taken as granted in Europe, North America, etc. In addition to the challenge of funding research in African countries, the pandemic paralyzed collaborations between dedicated individual researchers/groups and overseas partners. Nevertheless, the pandemic brought some interesting points of contact, interactions or synergies at UP. There are ongoing weekly town hall meetings where science and mathematical communities share their experiences on issues such as staff/student mental health, online teaching, assessment, invigilation, student success, research and postgraduate supervision.

Given these challenges and the complexity of the societal problem at hand, mathematicians have adopted the transdisciplinary research approach. We developed a model for the spread of COVID-19 in South Africa, see Garba et al. (2020). It is an extension of the SEIR model, modified by adding *A, J* and *P* respective classes of asymptomatic, isolated individuals and contaminated environment, thereby considering direct and indirect transmissions. The flow diagram of the model is given in Fig. 2.

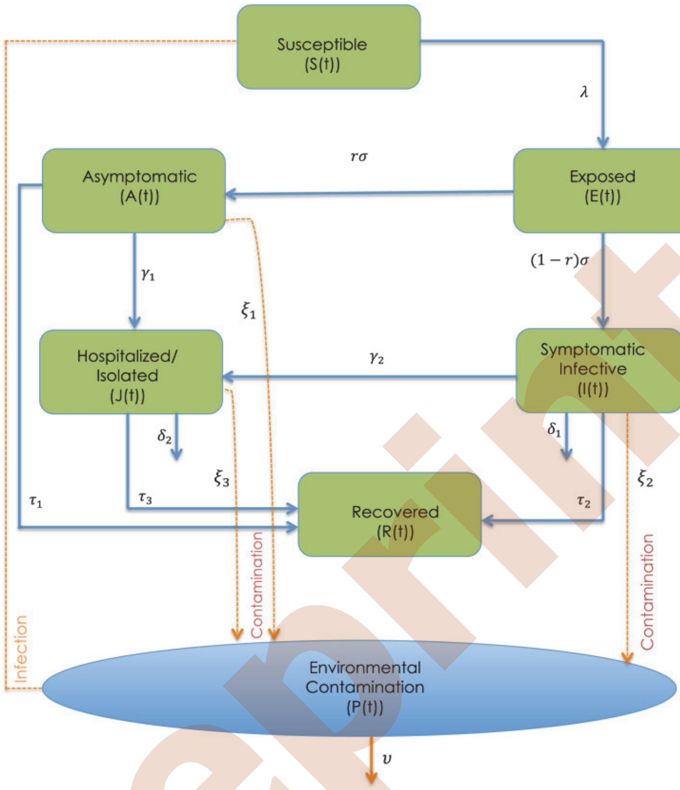

Fig. 1 Susceptible-Exposed-Asymptomatic-Symptomatic infective-Isolated-Recovered & Contaminated environment model

Our findings are summarized below. The model fitting with the number of deaths for the first six months of 2020 is excellent. We obtained reliable predictions in terms of peak times, numbers of cases and deaths. The continuum of disease-free equilibria is globally asymptotically stable when $\mathcal{R}_0 < 1$. Thus, the disease will eventually die out, particularly if Non-Pharmaceutical Interventions (NPIs) are implemented early and for a sustainable period of time. Further, the control reproduction number was estimated to 2.8 (3). Hence, 64 (67) % of the population should be vaccinated to achieve *herd immunity*, which is consistent with South Africa government predictions.

2.3. Conclusion

We initiated this work at the University of Pretoria. Since its emergence two years ago, COVID-19 has posed serious challenges, which led to some synergies between different mathematical communities as well as to transdisciplinary research (e.g., Google captures 36 700 000 mathematical articles). However, the challenges in teaching and research in Africa have been exacerbated by the lack of sufficient investment in this sector. These facts were echoed by the Heads of Governments at the Summit on Financing African Economies that was held in Paris on 18 May 2021. Therefore, mathematicians should speak out, advocate and make a call to the international community to invest in education and research in mathematics in Africa. Our ongoing research includes COVID-19 models with focus on multiple strains of the virus, comorbidity issues and vaccination intervention.

3. Challenges and Lessons Concerning Mathematics Education Brought by the Pandemic in the Latin American Context. *Panelist:* Nelly León Gómez

The pandemic has forced most teachers to make a very sudden transition to remote teaching; at present, mathematics teaching is still done remotely, or at best in some hybrid mode in many countries.

What are the principal challenges this situation has brought in your country, and more generally your region? How have these challenges been met? Which lessons for the future would you distill from this experience?

3.1. Introduction

The COVID-19 pandemic has plunged the entire world into a spiral of complex unprecedented situations that have affected the lives of human beings in all their dimensions: personal, family, social, emotional, economic. In particular, education has been greatly affected, especially in the less favored regions, as Latin America.

Below I will refer to some challenges faced and lessons distilled from this crisis in the Latin American region. These are supported by reports required from the national representatives of REDUMATE (Mathematics Education Network of Central America and the Caribbean), on the impact of the epidemic in schooling in our context; the policies, limitations and innovations to face this situation and to what extent families have been able to collaborate. I also followed the results of research and experiences published in the special issue of *Journal on Research and Teachers Preparation in Mathematics Education* entitled "Mathematics Education and the pandemic in the Americas" https://revistas.ucr.ac.cr/index.php/cifem/issue/view/3099

3.2. Principal challenges and how these challenges have been met

When the COVID-19 pandemic began in March 2020, one of the first actions taken in Latin America, as in the rest of the world, was to close schools and move to a distance education modality as a way to contain the spread of the virus. This measure had to be accompanied by others to guarantee school continuation, such as supporting food, health and the biopsychosocial well-being of children and young people. By abruptly switching to distance learning, I believe the main challenge was *how to deliver the teaching of Mathematics to every student guaranteeing quality and equity of mathematics learning in non-presence contexts*. No one was prepared to deal with this challenging situation. Although each country implemented action plans to face the challenge, each according to its own social, political and economic reality, I would say that it has not been fully met in Latin America. Multiple factors have had an impact on the development of these plans. These include the following:

3.2.1. Availability of technological resources

The conditions of accessibility to technology have meant a serious limitation for remote education in Latin America. A significant number of students and teachers do not have access to a computer, smartphone and internet connection, and many of them do not even have a TV set. According to the World Bank (2021) at the beginning of the pandemic, less than 43% of primary schools and less than 62% of secondary schools in Latin America had access to the internet for educational purposes. Among the online distance learning modalities, virtual asynchronous learning platforms were the most prevalent in the region of Latin America and the Caribbean; only 4 of the 29 countries offered live classes. (CEPAL – UNESCO, 2021) In addition, other distance learning solutions were deployed to bridge the gaps between schools and learners, such as broadcast educational programs by more traditional mass media such as radio or television, audios and videos (WhatsApp, e-mail) and printed materials.

3.2.2. Teacher's preparation and their willingness to take on the challenges of non-face-to-face mathematics education.

There is evidence of a lack of teacher preparation for an efficient application of technological tools in distance learning (beyond the use of these tools to maintain communication between students and teachers); in addition, artifacts and methods were used that were not entirely appropriate to the didactic transposition and the assessment of mathematics in this setting. Besides, some teachers feel that preparing a non-face-to-face Math class requires extra time, which they do not have because, in addition to teaching, they must do other tasks to supplement their low wage income.

3.2.3. Curricular adjustments

The adoption of distance or blended teaching modalities has created the need to identify key points of the curriculum on which to focus the attention of educational action. Content prioritization was necessary; consequently, the coverage of the actual mathematics curriculum is far from the expected standards, especially in public schools. This will cause the gap in mathematical skills between children from lower and higher socioeconomic background to get wider in Latin America.

3.2.4. Engagement of parents and families

Family support has been key to guaranteeing the continuity of education during the pandemic. In a depressed socio-economic context, numerous parents have faced difficulties in terms of their abilities and availability to support their children in learning mathematics and in the use of technology. Apart from family support, in some cases the children themselves have had to do informal work to help with the household economy leaving aside their studies, increasing the risk of dropout (León, 2021)

3.2.5. Motivation and emotional issues

The lack of direct interaction between students and teachers has increased the risk of disengagement especially for students who do not have access to online education. The stress of confinement, living conditions, restrictions on entertainment activities with friends, and the increase in physical abuse have greatly affected the emotional health of students. For this reason, regional government plans, such as "Every Family a School" in Venezuela, include a component of psycho-emotional care for families (MPPE, 2020).

3.3. Lessons distilled from the pandemic

The experience of facing the COVID-19 crisis has left important lessons to take into account in the new post-pandemic educational reality.

Lesson 1: The educational world has been digitized to large extent; as a result better versus worse access to digital communication channels translates into more versus less educational quality, inclusion and equity. Therefore, a strong investment in education is required in Latin America to address the technological gap. Such investment must be aimed not only at the equipment itself and at the improvement in connectivity, but also at the initial and on-service preparation of teachers to face the challenge of designing online tasks to engage students with mathematical content, while making use of such technological resources.

Lesson 2: It is important to capitalize on the technological push in education and the alternative forms of remote education developed during the pandemic. The use of digital technologies has generated new ways of thinking and representing mathematics, its teaching and assessment. The pandemic has also left us with a wide variety of innovative virtual resources created to enhance the teaching and learning of mathematics, focused not only on content but also on skills and competencies.

Lesson 3: Clear guidelines need to be established for the review of school contents and approaches in Mathematics in order to identify the mathematical knowledge and skills that will actually be required in the post-pandemic reality and to cover the knowledge gaps stemming from school closure, taking into account both the depth and variation in Math learning loss. This will take time and it will be necessary to act with prudence taking into account issues of equity and social justice in making quality mathematics education available to all.

Lesson 4: The classroom is not the only place for learning Mathematics. The closure of schools has generated interesting changes in the interaction model and has opened a range of possibilities that should be exploited to supporting students' independent and significant learning of Mathematics. Homeschooling is an experience not to be dismissed, since the new forms of relationship between school, family and community have led them to assume shared responsibilities regarding children's learning, and this must continue going on.

Lesson 5: No matter the form (in-person, remote or hybrid), teachers have to find the ways of teaching Mathematics that take into account the social and emotional needs of students, in order to maintain or to raise the expectation in their students, before launching into mathematical content.

4. Roles and Responsibilities of Mathematics Educators. *Panelist:* David Wagner

You are a researcher in mathematical education. How would you describe the role and the responsibilities of the mathematical education research community in the context of the current pandemic?

You are co-editor of a special issue of the journal Educational Studies in Mathematics centered on the pandemic and its challenges. What are the main messages you have taken away from this experience?

This pandemic has taught me that the responsibilities of mathematics educators in times of crisis should consider our complex arrays of responsibilities, from local, immediate challenges to the big questions. We need to devote time to love the people in our family and communities. We need to recognize the physical suffering, the isolation, and the compelling demands to care for others in new ways. In our teaching roles, we help students achieve their immediate needs, even when we think those needs are crazy—crazy demands from a crazy society. In our research, we study the local, immediate needs but also look at the big questions, and examine the structure beneath the crisis—the invariant things.

The coronavirus is probably not the most significant destructive force of our era. The social fabric of our world, the power structures we humans have erected and maintained, and our deep manipulation of our physical environments have been more destructive. These forces probably set the stage for the virus to be born, and certainly for the virus to multiply as it has and for the social chaos that ensued. The pandemic is an entanglement of the virus, the socio-political vectors and the environmental landscapes, all acting together. Thus, to answer the question about the responsibilities of mathematics educators in this pandemic, I step back to generalize and consider our responsibilities in crisis. We knew crisis was upon us before the pandemic. In 2013, crisis theorists Topper and Lagadec pointed to the human environmental footprint and the increasingly interconnected world and concluded: “major events are not new, but they have got denser” (p. 6). We live in a volatile world. Mandelbrot applied fractal geometry to volatile financial markets (Topper & Lagadec, 2013). Using this approach in the current crisis I see that we have to look through the massive changes and upheaval to examine what lies beneath—structures that have not changed.

When I consider the 161 articles received from our call for papers for the special issue on the pandemic in *Educational Studies in Mathematics* (Chan et al., 2021), and the thoughtful reviews of these papers, I see mathematics educators in action, responding to crisis. My co-editors, Man Ching Esther Chan and Cristina Sabena, and I saw that the pandemic has challenged usual patterns of research in our field. Many scholars were pressed with other demands and unable to focus on their research plans or on their reviewing commitments. Others found themselves with more time than before. Not surprisingly, the disparities appear to align with and magnify existing disparities such as gender disparities. This is an example of something that stays the same while we feel like everything is changing.

We saw scholars looking for accessible data that would help the field understand the pandemic. This was not easy. The pandemic makes it hard to start new studies involving participants because human interaction has been restricted. This is a major problem because we know that studies of social structures really need researchers to listen to the people most impacted by the structures. Over time, I expect that we will see more research that uses data that is harder to access, with deep engagement with the people most impacted by the crises.

Some researchers contributing to the special issue saw the pandemic as a prompt for questioning school curriculum. To identify what mathematics is needed by citizens to make sense of the crisis, researchers looked to public dialogue for analysis. We have seen governments, citizens and special interest groups disseminating graphs and statistics to explain pandemic events and inform action. Some of the research groups asked what mathematics is needed to make sense of these statistics (e.g., Kwon et al., 2021). We see that this does not exactly answer the question of what curriculum is needed. This is because the forms of statistics representation chosen by governments and others is guided by what they think citizens will understand. There is circularity when people ask what mathematics to teach based on what mathematics is being used. We need our field to identify new priorities for school mathematics based on analysis of the major challenges faced by society and individuals in the current age.

Some mathematics teachers have seen the pandemic as a prompt to re-examine their teaching. Surely, students should not accept a focus on the usual skills and knowledge when the world's habitual ways have clearly spelled catastrophe. One should expect that supposedly powerful mathematics would be used in class to address the most obvious disruption of our era. I would expect a call from students and from society, echoing the decades of injunction from Ubiratàn D'Ambrosio to examine the complicity of mathematics in the structures that allowed the virus to thrive in addition to the possibilities for using mathematics for justice in these times. However, speaking from my own experience and conversations with teachers, I see people distracted from asking deep questions—distracted by our social systems and the immediate needs of disrupted networks. Students and teachers focus on their compelling, immediate, local needs. While many are distracted, some educators are asking the bigger questions and trying to take the crisis seriously with their mathematics teaching. The special issue received articles that tell of ways mathematics teachers have addressed disease spread by studying different mathematics (Maciejewski, 2021), mathematics teachers' changing conceptualizations of their teaching due to isolation restrictions (e.g., Albano et al., 2021), and analysis of historical conversations about vaccination (Gosztonyi, 2021).

Many of the manuscripts we received studied distance teaching via digital and other technologies. This has been studied in our field for decades but now more scholars are interested. Borba (2021) reminds us that technologies of distance learning need to be seen as entangled with their contexts. One thing that is immediately clear in pandemic teaching is the inequities—unequal access to internet, unequal access to computers and tablets, unequal home infrastructure for uninterrupted time, unequal competing demands for time. Even while teachers and school systems work very hard at combatting them, these inequities persist. This is another example of something that is invariant in this time of massive change. We see that rural families have greater challenges (Yılmaz et al., 2021), and the needs of Indigenous students (Allen & Trinick, 2021), students of colour (Matthews et al., 20221) and students who have recently migrated are ignored in this crisis. The effects of poverty are magnified.

So I ask again, what are our responsibilities as mathematics educators? To answer this question, we need to answer other questions. What should every citizen know? Surely the answer is different than it was thirty years ago, considering the massive changes in interconnectivity in our world. To particularize this question, we need to identify the human and social problems of our time: What mathematics is necessary to understand interconnectivity? What mathematics is needed to understand climate? What mathematics is needed to understand biodiversity? What mathematics is needed to understand wealth distribution? With such questions I see deeper question: Should school mathematics focus on learning the useful algorithms when they can be performed instantaneously on handheld devices that are ubiquitous, or on applications to actual human problems? I know that this is an open question for many people, but I suggest that the public's evidently poor understanding of the science and mathematics of the pandemic may lead us to question the value of focusing school mathematics as we have in the past on procedural skills.

5. What Challenges Has the Pandemic Raised for Mathematics Education? *Panelist: Timothy Gowers*

The pandemic has put mathematics in the spotlight, and the media have solicited mathematicians, even those not experts in epidemiology. You have long been committed to communicating mathematical concepts to a wide audience.

Can you comment on challenges that are specific to the pandemic context?

What can we learn from initiatives realized by the mathematical community?

It is obvious that mathematics and the pandemic are closely intertwined. There is an entire academic discipline, epidemiology, devoted to the study of how diseases spread, and a significant component of that discipline is mathematical.

It is perhaps slightly less obvious that the pandemic raises challenges for mathematics education, but after a moment's reflection one can identify two important ones, at least if one takes "education" in a broad sense that moves beyond the confines of the classroom and takes in more general dissemination of mathematical ideas. The first is to improve mathematical understanding in society at large, so that people are better able to judge the reasons for and likely effects of the painful restrictions that have been imposed. The second is to improve the mathematical understanding of the principal decision makers, so that the decisions they make, which can have huge consequences, are made as rationally as possible. These two challenges are closely related, since it is much easier that politicians will make the right decisions if they can count on public support and understanding.

5.1. Public understanding of pandemic-related mathematics

Over the last two years, many people have made extraordinary sacrifices. Some have argued that these sacrifices were largely unnecessary, or at least that the benefits were outweighed by the costs. And while it is certainly right to weigh up the costs and benefits, to do so properly requires an appreciation of a few basic mathematical principles.

5.1.1. Exponential growth

To many non-mathematicians, the word "exponentially" is little more than a synonym for "quickly". So if they are told at the early stages of a pandemic that the case numbers are growing exponentially, but after two or three weeks the numbers are still small, they may wonder whether they have been misled. A more serious problem is that it is very hard to persuade people to accept significant restrictions while numbers are still small. The argument for doing so is that the earlier one imposes restrictions, the less time it takes to reduce case numbers to a level where the restrictions can be relaxed again, and the less illness and death there will be as well. But to understand this properly, one needs mathematics – not the sophisticated mathematics of a professional epidemiologist, but just the basics of exponential growth. Without such an understanding, one may be swayed by arguments such as "More people were killed by lockdowns than by Covid." In many countries that is clearly untrue, but in a country such as New Zealand, which had several lockdowns and very few COVID-19 deaths, it is almost certainly true. Does that mean that New Zealand made a big mistake? No, because the relevant comparison is between the number of deaths caused by lockdown and the number of COVID-19 deaths that there would have been without lockdown.

The point I am making here is not so much the arguments for and against lockdowns and other restrictions. Rather, I am arguing that greater public understanding of mathematics can lead to a greater willingness to accept policies that are for the good of everybody, but which are somewhat counterintuitive.

5.1.2. The role of data and modelling

Mathematical modelling is of central importance in epidemiology, but the role of modelling is not well understood by the general public. If a simulation is run on a computer, for example, it will be based on assumptions, which themselves will be based on data that is usually uncertain and incomplete, especially in the early days of the pandemic. Thus, predictions are typically conditional, but they are often presented as unconditional by journalists. Furthermore, models can be self-defeating in the following sense: if a model makes an alarming prediction, politicians may well impose restrictions that stop the alarming prediction from coming to pass.

These phenomena, which are absolutely normal and expected, have led in the UK to considerable public distrust of modelling as a discipline. This is a problem not just for managing the pandemic, but also for other areas where modelling meets public policy, an obvious example being climate change.

5.1.3. Probability and risk

Another aspect of policy making that was underappreciated by the general public was the role of uncertainty. In the early stages of a rapidly developing pandemic, decisions had to be made quickly when many facts were unknown, of which the most important was how COVID-19 spread. When one is weighing up the costs and benefits of a possible decision under these circumstances, they necessarily come with a probability attached, or more precisely a probability distribution.

A simple example of this was the question of whether mask mandates were a good idea. Early on in the pandemic, the evidence for beneficial effects of mask wearing (mainly in protecting others from the wearer) was weak. To a non-mathematician, it might seem an obvious consequence of this that there was only a weak case for encouraging the wearing of masks. However, because of the nature of exponential growth, a small reduction in the growth rate is hugely beneficial, whereas the cost of widespread mask wearing is small. So even if the reduction in the growth rate was not certain to occur, the expected net benefit of mask wearing was large.

5.2. Trust in science

One of the great potential benefits of better understanding of mathematics among the general public and politicians would be a healthier relationship with scientists. I have given several examples already of how a lack of understanding of mathematics can lead people to lose trust in science, and this has been a serious problem.

Another problem, which I have not yet mentioned, concerns the relationship between politicians and scientific advisors. A politician with a good mathematical understanding can understand not just the advice but the justification for the advice; a politician without it simply has to accept or reject the conclusion. It was extraordinary as a British person to see Angela Merkel, who has a scientific background, giving a beautifully clear explanation of exponential growth and how it informed her decisions, and to contrast that with Boris Johnson, who does not have a scientific background, simply saying in a vague way, "We are following the science," (though he said that less as the pandemic proceeded).

5.3. What can we do?

While one can see the benefits that improved mathematical understanding would have, it is less clear how any improvement can be achieved. The raising of standards of mathematical literacy is of course a central aim of mathematical education, so to call for it may seem a little pointless. However, my conclusion is a little more specific, since some parts of mathematics have a much bigger effect on the quality of public decision making than others. The basics of probability and statistics, for example, are clearly important for conducting risk-benefit analyses, whereas an understanding of polynomials, while essential for any technical uses of mathematics, is less necessary for public appreciation of political decisions.

So a potential way forward is to identify those areas of mathematics that would be most helpful for improving public discourse and decision making, and to think of creative ways of explaining them to non-expert audiences. That is still a big challenge, and maybe for many people it is simply too late to get them interested. But at the very least one could think about how best to bring up a new generation to be better educated in these aspects of mathematics than the current generations are. A natural idea to try is to design school mathematics courses that are principally aimed at people who will not be specializing in STEM subjects. Such courses could analyze current events from a mathematical perspective, giving people the tools to think about them more effectively. A well designed course of this kind would have the potential to demonstrate that mathematics is, to use words of Jordan Ellenberg (2014) "like an atomic powered prosthesis that you attach to your common sense, vastly multiplying its reach and strength". In that way, it might appeal to people who would otherwise remain unaware of its benefits to individuals and to society.

6. Reflections and Lessons

We often complain that despite their crucial role in our technological societies, mathematics remains invisible. This is no longer the case. The COVID-19 pandemic has shown the importance of mathematical models for understanding the course of the pandemic, anticipating and weighing possible consequences of different policy decisions, or managing and analyzing the very large volumes of data collected. It has shown the potential but also the limits of these models, the need to constantly adjust them due to the emergence of variants, the effect of decisions taken, etc. It has confronted the general public with science in the making, made up of questions and doubts, as opposed to the image of certainty that many had formed during their schooling. It has shown the need for a solid and shared mathematical and scientific culture within our societies, a culture that is largely lacking as shown by the permeability of the population to the incredible fake news stories that have multiplied.

Furthermore, the sudden shift to distance and, at best, hybrid teaching has profoundly destabilized education systems that were totally unprepared for it. Educational inequalities have been exacerbated between countries and between pupils within the same country. The challenges are immense!

The panel has made clear that, faced with these challenges, the mathematics and mathematics education communities have mobilized strongly, each with its own expertise, means and fields of action. They have mobilized at the level of research and practical action. In addition to epidemiological mathematicians, such as Jean Lubuma, who are directly involved in research on this pandemic, many mathematicians, as Timothy Gowers has shown, have helped the general public and the media to make sense of epidemiological models, of the growth processes and probabilistic reasoning modes involved. The world of mathematics education has also reoriented its research to meet the challenges encountered, as David Wagner has shown, and beyond research, it has invested heavily in the production and sharing of online resources for teaching and training. Solidarities have been strengthened or created, as Nelly León has shown.

From this point of view, the panel also carries a message of hope. This message is all the more necessary as the current pandemic is not an isolated crisis. We will face, and are already facing, other crises, undoubtedly even more serious and lasting, such as those associated with climate change. The need for quality mathematical and scientific education for all is essential, and taking up this challenge requires the synergy of the strengths of the mathematical community at large.

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